

Primer on Probability and Odds and Interpreting Their Ratios

Chance is measured by using either probabilities (a ratio of occurrence to the whole) or odds (a ratio of occurrence to non-occurrence). Consider measuring the chance of breast-feeding among 1000 new mothers. If 600 ultimately breast-feed, the prob-

ability of breast-feeding is 600/1000, or 0.6 (often expressed as 60%), whereas the odds of breast-feeding are 600/400, or 1.5 (often expressed as 1.5 to 1). Table 1 summarizes the characteristics of probability and odds.

TABLE 1

CHARACTERISTIC	PROBABILITY	ODDS
Ratio	$\frac{\text{occurrence}}{\text{whole}}$	$\frac{\text{occurrence}}{\text{nonoccurrence}}$
Range	0 to 1	0 to ∞
Transformation to other measure	$\text{odds} = \frac{\text{probability}}{1 - \text{probability}}$	$\text{probability} = \frac{\text{odds}}{1 + \text{odds}}$

Probabilities and odds contain the same information and are equally valid measures of chance. In the case of infrequent events (i.e., probability < 0.1 or 10%), the distinction is unimportant (probability and odds have essentially the same value). However, as shown in Table 2, probability and odds take on very different values as the chance of an event increases.

Although probabilities are often reported in the medical literature, it is rare to see odds reported. On the other hand, ratios of probabilities (i.e., relative risks, or risk ratios [RRs]) and odds (i.e., odds ratios [ORs]) are seen often. And it is in these ratios of ratios that the distinction between probability and odds may be both important and ambiguous.

When the chance of common events are being compared, ORs and RRs substantially diverge in value. Let's return to the breast-feeding example. Imagine a randomized trial of a lactation-support system. The probability of breast-feeding in the control group is 60% (or an odds of 1.5); in the intervention group, it is 90% (or an odds of 9). Table 3 shows that the relative risk is 1.5 while the odds ratio is 6.

TABLE 2

PROBABILITY	ODDS
1	∞
0.9	9.00
0.8	4.00
0.7	2.33
0.6	1.50
0.5	1.00
0.4	0.67
0.3	0.43
0.2	0.25
0.1	0.11
0	0.00

TABLE 3

GROUP	PROBABILITY OF BREAST-FEEDING	ODDS OF BREAST-FEEDING	RELATIVE RISK (INTERVENTION VS. CONTROL)	ODDS RATIO (INTERVENTION VS. CONTROL)
Control	0.6	1.5	0.9/0.6	9.0/1.5
Intervention	0.9	9.0	=1.5	=6

In general, ORs are more extreme (i.e., farther away from 1) than are RRs. ORs that are greater than 1 exaggerate the increase in risk (i.e., OR > RR); ORs that are less than 1 exaggerate the decrease in risk (i.e., OR < RR). Practically speaking, the discrepancy between the two measures is relevant only when relatively common events are being compared. Readers should begin to worry about the distinction when baseline probabilities exceed 10% to 20%. And, as shown in Table 4, they might reasonably pursue a conversion when baseline probabilities are greater than 50%.

It is important to emphasize that ORs and RRs are equally valid—but different—measures. Readers are seeing more and

more ORs in the medical literature, largely because of the increased use of logistic regression. Because most people are more familiar with probabilities than odds, ORs are often interpreted as RRs. When events are common, this misinterpretation substantially exaggerates the association being reported. If the goal is clarity, the probability (or absolute event rate) for each group is tough to beat.

Suggested Reading

Talryn H, Davies O, Crombie IK, Tavakoli M. When can odds ratios mislead? *BMJ*. 1998;316:989-91.

Zhang J, Yu KF. What's the relative risk? A method of correcting the odds ratios in cohort studies of common outcomes. *JAMA*. 1998;280:1690-1.

TABLE 4

APPROXIMATE RELATIVE RISK FOR ODDS RATIOS GREATER THAN 1						APPROXIMATE RELATIVE RISK FOR ODDS RATIOS LESS THAN 1							
		BASELINE PROBABILITY							BASELINE PROBABILITY				
		50%	60%	70%	80%	90%			50%	60%	70%	80%	90%
ODDS RATIO	1.2	1.09	1.07	1.05	1.03	1.02	ODDS RATIO	0.9	0.95	0.96	0.97	0.98	0.99
	1.5	1.20	1.15	1.11	1.07	1.03		0.8	0.89	0.91	0.93	0.95	0.98
	2	1.33	1.25	1.18	1.11	1.05		0.5	0.67	0.71	0.77	0.83	0.91
	5	1.67	1.47	1.32	1.19	1.09		0.3	0.46	0.52	0.59	0.68	0.81
	10	1.82	1.56	1.37	1.22	1.10		0.1	0.18	0.22	0.27	0.36	0.53